

KLOOSTERMAN CRYSTALS FOR REDUCTIVE GROUPS

I will first review the relationship between the classical Bessel equation

$$\left(x \frac{d}{dx}\right)^2 u - xu = 0,$$

and the classical Kloosterman sum

$$\mathrm{Kl}(\mathbf{a}) := \sum_{xy=\mathbf{a} \in \mathbb{F}_p} \exp\left(\frac{2\pi i}{p}(\mathbf{x} + \mathbf{y})\right).$$

Such a relation can be regarded as an instance of the geometric Langlands correspondence for GL_2 . I will survey the recent generalizations of this story for arbitrary reductive groups, based on the works by Frenkel-Gross, Heinloth-Ngô-Yun, and X. Zhu. In the end, I will report the joint work in progress with X. Zhu, where we study the p -adic aspect of this theory.