Local Models For Moduli Of Global and Local G-Shtukas

Esmail Arasteh Rad
Univ. Münster

July 22, 2020

Outline Of The Talk

Outline Of The Talk

- Beginning
- Middle
- End

xShimura data and r H-data

xShimura data and r H-data

- A Shimura datum $(G; X; K)$
  - a reductive group G over Q with center Z,
  - G(R)-conjugacy class X of homomorphisms S ! G_R for the Deligne torus S,
  - A compact open sub-group K G(A_f),

*x*Shimura data and *r H*-data

- A Shimura datum $(G; X; K)$
  - a reductive group G over Q with center Z,
  - G(R)-conjugacy class X of homomorphisms S ! G_R for the Deligne torus S,
  - A compact open sub-group K G(A_f),
  subject to certain conditions...

xShimura data and r H-data

- A Shimura datum $(G; X; K)$
  - a reductive group G over Q with center Z,
  - G(R)-conjugacy class X of homomorphisms S ! G_R for the Deligne torus S,
  - A compact open sub-group K G(A_f),
  subject to certain conditions...

$$(G; X; K) \quad Sh_K(G; X) = G(Q)nX \quad G(A_f)=K$$

- Let's wish a modular interpretation exists...

- Let's wish a modular interpretation exists...

  \In order to be able to realize all but a handful of Shimura varieties as moduli varieties, we shall need to replace algebraic varieties and algebraic classes by more general objects, namely, by motives" (Milne 198?)

- Let's wish a modular interpretation exists...

  \In order to be able to realize all but a handful of Shimura varieties as moduli varieties, we shall need to replace algebraic varieties and algebraic classes by more general objects, namely, by motives" (Milne 198?)

  inspired by a former observation of P. Deligne

  \Pour interpreter des structures de Hodge de type plus complique, on aimerait remplacer les varietes abeliennes par des \motifs" convenables, mais il ne s'agit encore que d'un rêve." (Deligne 1979, p. 248)

xThe Journey to the Dreamland of FF

Category Of Motives / FF

xThe Journey to the Dreamland of FF

Category Of Motives / FF

$$Q \; ! \quad \text{smooth proj. curve} C = \mathbb{F}_q$$

Remark (Functoriality)

The assignment

$$(C;\mathsf{G}) \nrightarrow r_n H^1(C;\mathsf{G})$$

is functorial on $(C;\mathsf{G})$. In particular $\varphi : \mathsf{G} \to \mathsf{G}'$ induces

$$\varphi : r_n H^1(C;\mathsf{G}) \to r_n H^1(C;\mathsf{G}'):$$

Remark (Functoriality)

The assignment

$$(C;\mathsf{G}) \mapsto r_n H^1(C;\mathsf{G})$$

is functorial on $(C;\mathsf{G})$. In particular $\varphi : \mathsf{G} \to \mathsf{G}'$ induces

$$\varphi : r_n H^1(C;\mathsf{G}) \to r_n H^1(C;\mathsf{G}'):$$

which is induced by

$$\varphi : H^1(C;\mathsf{G}) \to H^1(C;\mathsf{G}'),\ G \mapsto G \times^{\mathsf{G}} \mathsf{G}':$$

between the stack of G-bundles and G'-bundles over C.

xRealization Functors I

- Definition (Étale realization)

xRealization Functors I

De nition (Etale realization)

Fix $\_ = (\ _i) \ 2 \ C^n$. Let $A_{\_}$ denote the completion of O_{C^n} at $\_$ and let $r\ _nH^1(C;G)_{-} = r\ _nH^1(C;G) \quad _{C^n} \mathrm{Spf} A_{\_}$. Assume that S is connected, x a geometric base point of S.